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The breakdown of topology at small scales

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Abstract: We discuss how a topology (the Zariski topology) on a space can appear to break down at small distances due to D-brane decay. The mechanism proposed coincides perfectly with the phase picture of Calabi-Yau moduli spaces. The topology breaks down as one approaches non-geometric phases. This picture is not without its limitations, which are also discussed.

Keywords: D-branes, Superstring Vacua.

Contents

1. Introduction

The concept of spacetime is supposed to be derived rather than fundamental in superstring theory. Many ideas have emerged concerning how one might obtain such a derivation but none seems to be without its flaws.

To avoid the extraordinary complications introduced by the presence of a nontrivial time-like direction, it is simplest to study a compactification model. Here one can ask how to deduce the geometry of a Calabi-Yau space X of complex dimension d used to compactify string theory down to $10 - 2d$ dimensions. Then one can ask to what extent the geometry of X can be deduced from "intrinsic" properties of the string, which in many cases can be paraphrased in terms of the physics of the uncompactified dimensions.

It is a repeated phenomenon in string theory that questions associated to compactifications tend to be most mathematically interesting in the case $d = 3$ which happily coincides with observed 4-dimensional spacetime. It is this case that we consider here.

Let us consider a type-II string compactification. The D-branes corresponding to BPS solitons of the resulting $N = 2$ theory are the focus of our attention. Following the work of Kontsevich [1] and Douglas [2], it is known that some of the D-branes (the "B-branes") are described by the derived category of coherent sheaves on X. In this context, the conditions for stability [3, 4, 5] are now understood to a fair extent. In particular, it is known that many D-branes which are understood to exist in flat space become unstable as the characteristic radius of a Calabi-Yau becomes small with respect to the scale set by the string tension.

In this short paper we outline a simple idea for how one might use such a decay of D-branes to motivate the notion that topology is only well-defined in the large radius limit. We should immediately concede that this argument is not without its limitations not least

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of which is our use of the Zariski topology. This topology is not usually associated with physics. The main point we wish to emphasize is that the picture we propose coincides beautifully with the "phase" picture of [6, 7]. At least in the context of linear σ-models or, equivalently, toric geometry, the moduli space of $N = (2, 2)$ superconformal field theories can be roughly divided into phases, each with a particular interpretation. The "geometric phases" consist of the smooth Calabi-Yau X together with other phases representing singular versions of X such as orbifolds. The "non-geometric phases" consist of interpretations such as Landau-Ginzburg models, fibrations with a Landau-Ginzburg fibre ("hybrid models") and exoflops which have a Landau-Ginzburg model associated to some subspace. In the language of algebraic geometry, these Landau-Ginzburg models can be associated with non-reduced schemes.

We will see that the topology is lost as one moves from the geometric phases to the non-geometric phases as one would hope. In [8, 9] the author hoped that 0-branes might play the key rôle in this loss of geometry or topology. Indeed, it is 0-branes that decay as one passes through a flop [10]. However, in non-geometrical phases the 0-brane remains stubbornly stable [4, 11]. Instead, as realized in [2, 12], it is the 4-branes which naturally decay along the phase boundaries. Our purpose here is to argue that this can be associated with a loss in topology and that it is a general feature of all models associated to toric geometry. So the picture appears to be that 0-brane decay is associated with a "change" in topology while 4-brane decay is associated to a "loss" of topology.

2. The Zariski topology

Let X be a topological space. That means that X is a set of points and we have a set of subsets of X, called "open sets", satisfying the following conditions:

1. X itself and the empty set $\emptyset$ are both open.
2. Any union of open sets is open.
3. Any finite intersection of open sets is open.

If X has a metric, then one can form a topology with open sets consisting of unions of interiors of balls of any radius $\epsilon > 0$ and any center. This "metric topology" is probably the one we are most used to thinking about in physics.

In algebraic geometry, the natural topology is the "Zariski topology" defined as follows. Let X be embedded in $\mathbb{P}^N$ as the intersection of the zeroes of some algebraic equations in the homogeneous coordinates of $\mathbb{P}^N$. If f is any algebraic equation in the homogeneous coordinates of $\mathbb{P}^N$, then the intersection of $f = 0$ with X will be an "algebraic" subspace of X (typically of codimension one). Let U_f be the *complement* of this subspace in X.

The Zariski topology on X is then defined as the open sets consisting of $\emptyset$, X, unions of U_f's, and finite intersection of U_f's, for any f's. In other words, the open sets consist of complements of algebraic subspaces of X.

Note that the Zariski topology can be built from a knowledge of the algebraic codimension one subspaces, i.e., *divisors* of X.

It should be admitted from the outset that the Zariski topology is peculiar by physicists standards. For example, it is never Hausdorff in nontrivial examples. The problem is that it is very "coarse", i.e., it consists of relatively few open sets. Having said that, for some purposes (e.g., computing cohomology) it is equivalent to a metric topology. This relationship was explored in the famous GAGA paper [13].

Knowing to what extent the Zariski topology suffices for string theory is probably a deep question and we will certainly not attempt to answer it here. Let us just suggest that many approaches to string theory are algebraic in nature, and in such a context one should expect something like the Zariski topology to be very natural.

3. D-branes

The Zariski topology suggests that we are interested in the following D-branes on the Calabi-Yau threefold X:

1. The 0-branes which form the "points".

2. The 4-branes which form the divisors.

These are all B-branes. That is, they are BPS branes which are preserved by the twist to the topological B-model.

In this paper we will restrict our attention to the zero string-coupling limit. In this case it is established that B-branes are described by the derived category of coherent sheaves on X [1, 2, 14, 9, 15, 16].

The 0-branes consist of (a complex with a single nonzero element given by) a skyscraper sheaf $\mathscr{O}_p$ for $p \in X$. Similarly the 4-branes are associated to the sheaves $\mathscr{O}_D$ supported on divisors $D \subset X$.

It is an interesting question to ask to what extent one might determine the set of 0-branes given only the purely categorical (i.e., worldsheet) description of the D-branes. This was discussed in the context of string theory in [10] based on the work of Bondal and Orlov [17]. Here we will assume that a determination of the 0-branes and 4-branes has been made.

Given a 0-brane corresponding to p and a 4-brane corresponding to D, there is an open string between these D-branes in the topological sector precisely when $p \in D$. Thus the topological field theory has enough information to define the Zariski topology. In derived category language, the open set U_D, equal to the complement of D, is given by the set

$$U_D = \{\mathscr{O}_p \mid \operatorname{Hom}(\mathscr{O}_D, \mathscr{O}_p) = 0\}\,. \tag{3.1}$$

It was realized by Douglas [2, 12] that the 4-branes can decay as one moves away from the large radius limit. It is this observation that we will exploit here.

4. The quintic threefold

Let us quickly review a result from [4]. Suppose X is the quintic threefold and is given by the embedding $i : X \hookrightarrow \mathbb{P}^4$.

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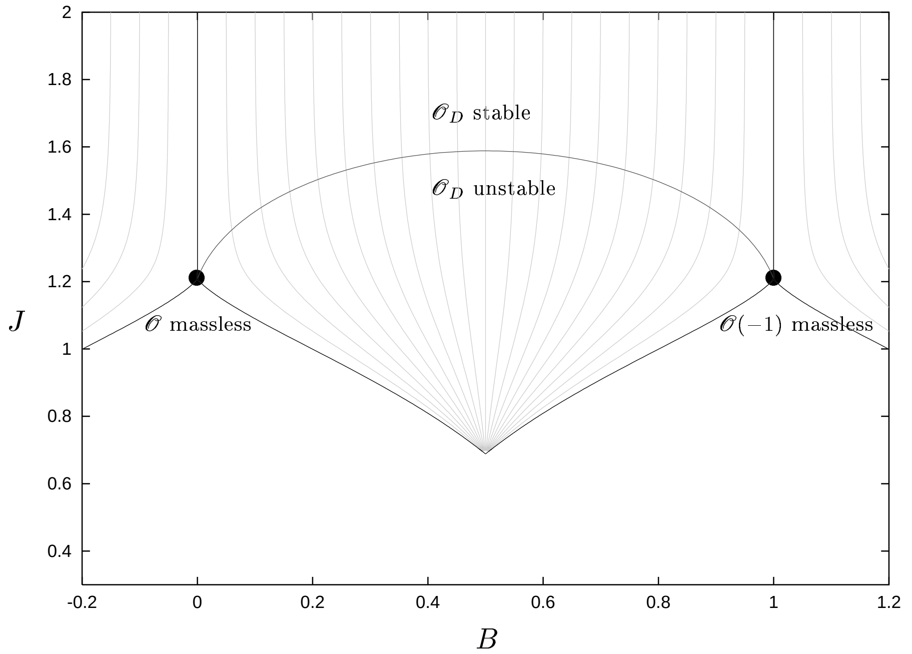

Figure 1: 4-brane decay on the quintic.

Let $\mathscr{O}(k)$ denote the pullback, via i, of the usual twisted sheaves on $\mathbb{P}^4$. We then have a short exact sequence

$$0 \to \mathscr{O}(-1) \xrightarrow{f} \mathscr{O} \to \mathscr{O}_D \to 0\,, \tag{4.1}$$

where D is a degree one divisor in X associated to the linear map f.

Ignoring other possible decays, the stability of $\mathscr{O}_D$ is determined by the mass of the open string associated with the map f. $\mathscr{O}_D$ is stable if and only if this open string is tachyonic. This mass was computed explicitly in [4] and the result is shown in figure 1. This figure shows the complex $B + iJ$ plane. This plane is divided into non-tesselating fundamental regions of the moduli space as discussed in [18]. At large radius $\mathscr{O}_D$ is stable. As we cross the line of marginal stability the 4-brane decays into $\mathscr{O}$ and $\overline{\mathscr{O}(-1)}$.

The line of marginal stability ends at the points where the decay products become massless. This is due to a general rule of Π-stability as argued in [4] — a line of marginal stability will either end at a point where a decay product becomes massless or it will end on another line of marginal stability for that decay product.

The decay products $\mathscr{O}$ and $\mathscr{O}(-1)$ become massless for $B = 0$ and $B = 1$ respectively. The line of marginal stability forms a "wall" across the fundamental region shown in the figure. This ties in with the "phase" picture for $N = (2, 2)$ superconformal field theories studied in [6, 7]. The 4-brane is stable while we are in the "Calabi-Yau phase" and becomes unstable as we pass into the "Landau-Ginzburg" phase.

So far we only considered divisors of degree one. One may also consider higher degree cases given by the exact sequence:

$$0 \to \mathscr{O}(-k) \xrightarrow{f} \mathscr{O} \to \mathscr{O}_{D_k} \to 0\,, \tag{4.2}$$

where D_k is a divisor of degree k. As discussed in [4], such a 4-brane decays in a similar way to the degree one case but will decay at a larger radius depending on k.

The picture we wish to present is then as follows. Near large radius limit, very nearly all of the 4-branes on X are stable, allowing us to determine the Zariski topology to a good approximation. Then as the radius of X decreases, we start to lose 4-branes of progressively lower degree. As we pass out of the Calabi-Yau phase, we lose the last 4-brane and the Zariski topology is completely lost.

5. The general phase picture

A general Calabi-Yau threefold X will have more than one deformation of $B+iJ$ and so the above picture of walls of marginal stability will be become considerably more complicated. Fortunately we know enough about general features of the moduli space to generalize the above computation.

Suppose that $H^2(X,\mathbb{Z})$ is generated by e_j, where $j = 1 \dots h^{1,1}(X)$. Suppose further that each e_j is positive in the sense that it is Poincaré dual to an algebraic 4-cycle $D_j \subset X$. These divisors D_j are the 4-branes whose decay we wish to analyze.

Assume the decay is associated to the obvious sequence:

$$0 \to \mathscr{O}(-D_j) \xrightarrow{f} \mathscr{O} \to \mathscr{O}_{D_j} \to 0\,, \tag{5.1}$$

where $\mathscr{O}(-D_j)$ is the line bundle on X with $c_1 = -e_j$. This implies that the wall of marginal stability will stretch between a locus where the basic 6-brane $\mathscr{O}$ becomes massless and a locus where $\mathscr{O}(-D_j)$ becomes massless. When these branes become massless, the associated conformal field theory becomes singular and we are on the discriminant locus within the moduli space of $B+iJ$. This is equivalent to the classical discriminant in the moduli space of complex structures of the mirror to X.

Except for the simplest of examples, an explicit computation of the discriminant is not practical. Suppose we restrict attention to the large "Batyrev-Borisov" class of complete intersections in a toric variety [19, 20]. Virtually all known Calabi-Yau threefolds fall into this class. Many features of the discriminant in this case have been studied by Gelfand, Kapronov, and Zelevinsky [21]. Much of this structure has also been analyzed directly from the linear σ-model by Morrison and Plesser [22].

The Calabi-Yau X is associated to a set of points $\mathscr{A}$ on a lattice N (see [23], for example, for more details). Let P_X be the convex polytope of this point set. The discriminant is reducible with each irreducible component associated with a face (of any codimension) of this polytope. One irreducible component is distinguished — namely the one associated to P_X itself. We call this the "primary component" of the discriminant which we denote Δ_0.

It has been conjectured [24, 25, 26, 8] that this primary component of the discriminant is precisely where the basic 6-brane $\mathscr{O}$ becomes massless. This is a very natural conjecture and has been shown to be true in several examples. We will assume it to be true.

The statement that a certain D-brane becomes massless for a certain component of the discriminant locus is not really well-defined. One must specify some basepoint in the moduli space (usually near large radius) to define the labeling of the set of D-branes and

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then specify the path taken to the discriminant. It follows that the above conjecture should really assert that, for some naturally defined short path from the large radius limit to the primary component of the discriminant, the D-brane $\mathscr{O}$ becomes massless.

Suppose we loop around the large radius limit point before we embark on this path towards the primary component Δ_0. Under the established rules (see, for example, [26, 4]) the monodromy around large radius limit gives an autoequivalence of $\mathbf{D}(X)$ given by $\mathscr{F} \mapsto \mathscr{F} \otimes L$ for some line bundle L. $c_1(L)$ is determined by exactly how one went around the large radius limit. In particular, if we perform this monodromy by varying the component of $B + iJ$ associated to e_j, we obtain $L = \mathscr{O}(-D_j)$. Thus the D-brane $\mathscr{O}(-D_j)$ also has vanishing mass on Δ_0 — we just have to loop around the large radius limit in the right way before heading off towards Δ_0.

The case of the quintic in figure 1 should help the reader visualize this construction. Here the primary component of the discriminant consists simply of the "conifold point". The D-brane $\mathscr{O}$ indeed becomes massless here but $\mathscr{O}(-1)$ becomes massless at an image of this point in the Teichmüller space under monodromy around the large radius limit.

We have therefore established a higher-dimensional analogue of figure 1. The wall of marginal stability will form a wall which encloses the large radius phase in the moduli space of $B + iJ$ in the direction of a given e_j. By considering each divisor D_j in turn we may completely wall off the large radius limit. The location of this (real co-dimensional one) wall is guided by the location of the (complex co-dimensional one) primary component of the discriminant locus Δ_0. Each one-complex-dimensional slice of the moduli space should look roughly like figure 1.

To understand a more complicated example consider the much-studied two-parameter model of [27]. Our notation is taken from [8]. Here we have four phases consisting of a Calabi-Yau, an orbifold, a hybrid and a Landau-Ginzburg theory. The hybrid consists of a fibration over $\mathbb{P}^2$ with fibre given by a Landau-Ginzburg theory. Projecting this moduli space into the "algebraic" J-plane as discussed in [28], we obtain figure 2. The discriminant locus has two components shown as Δ_0 and Δ_1 in the figure. The primary component, Δ_0, lies in 3 of the 4 walls forming the phase "boundaries". The wall between the Calabi-Yau phase and the orbifold phase contains only Δ_1.

Imagine passing from the Calabi-Yau phase to the hybrid phase. Since we cross Δ_0, all the 4-branes dual to that direction in $H^2(X)$ decay, thus destroying the Zariski topology.[1] Passing from the Calabi-Yau phase to the orbifold phase involves blowing down an exceptional $\mathbb{P}^2$. This does *not* involve crossing Δ_0 and so we do not expect a loss of topology. Indeed, the 4-branes which wrap the exceptional $\mathbb{P}^2$, which are the ones one might expect to decay, instead become massless on Δ_1. Proceeding further to the Landau-Ginzburg phase should cause theses 4-branes to decay.

We therefore have a Zariski topology apparently well-defined in the Calabi-Yau and orbifold limits of the theory, but not in the hybrid and Landau-Ginzburg limits. That is, we have a topology in the "geometric" phases as one might expect.

[1]The 4-branes wrapping the $\mathbb{P}^2$ base are still stable. One might use this to say that part of this model still has a geometrical interpretation as would befit a "hybrid" model.

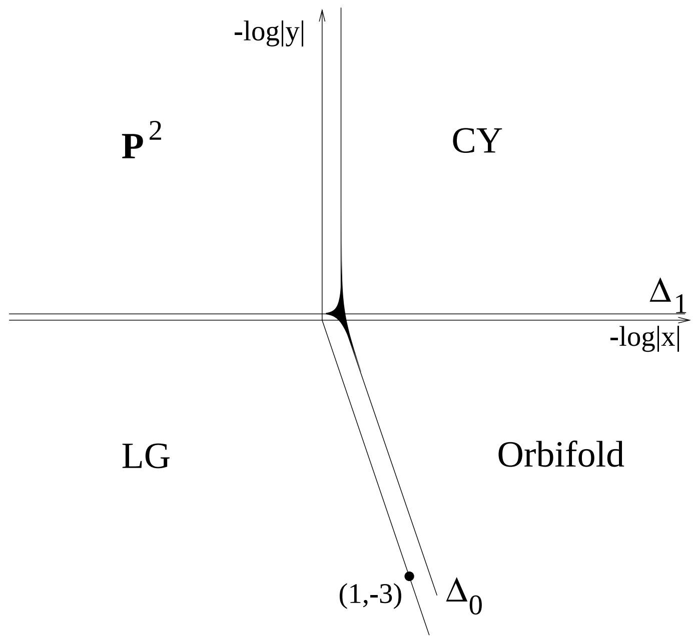

Figure 2: The 4 phases of the 2 parameter example.

One can define exactly what one means by "geometric phases" in the case of hypersurfaces in toric varieties by demanding that each simplex in the triangulation of $\mathscr{A}$ has the unique interior point as a vertex [7]. A generalization to complete intersections exists. As argued in [8], based on the results in [21], the phase boundaries between geometric and non-geometric phases always contain Δ_0 whereas the phase boundaries between two geometric phases never contain Δ_0. Therefore, *we always lose the Zariski topology as we leave the geometric phases.*

6. Problems

The reader may have noticed that we have an unnecessarily restrictive definition of a 4-brane. According to the usual correspondence between sheaves and bundles, the sheaf $\mathscr{O}_D$ corresponds to the trivial line bundle over D (extended by zero to form a sheaf over X).[2] Why not choose a nontrivial line bundle over D?

For example, in the quintic there is the 4-brane $\mathscr{O}_D(1)$ defined by the sequence:

$$0 \to \mathscr{O} \xrightarrow{f} \mathscr{O}(1) \to \mathscr{O}_D(1) \to 0\,. \tag{6.1}$$

The mode of decay of $\mathscr{O}_D(1)$ represented by this sequence forms a line of marginal stability between the points where $\mathscr{O}$ and $\mathscr{O}(1)$ become massless. In terms of figure 1, this would move the line of marginal stability by a shift of B by one unit to the left. Thus we *could* make this 4-brane D-decay by going around the large radius limit once before heading off to the Landau-Ginzburg phase. However, if we head straight for the Landau-Ginzburg phase this decay will not happen.

[2] This correspondence actually gets shifted in string theory. See [29] for more details.

It would be nice if we could show that all such 4-branes really decay by some other channel. The current understanding of Π-stability makes it very difficult to assess the stability of any D-brane except in the easiest of examples. Several obvious potential decay modes for $\mathscr{O}_D(1)$ were checked without success. However, given the lack of a systematic method for checking decay modes, this does not constitute evidence that the D-brane is stable.

One might try to find the stable D-branes at the Gepner point directly using boundary conformal field theory methods such as in [30, 31, 32]. It appears to be a formidable task to completely describe all the possibilities but there appears to be no stable state with the charge of a 4-brane (with any number of 2-branes added) of the limited types of D-brane studied in [30, 31].

An observation which might count against the decay of $\mathscr{O}_D(1)$ comes from the analysis of the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold. Let $E \cong \mathbb{P}^2$ be the exceptional divisor in this case and let $C \cong \mathbb{P}^1$ be a complex line on E. The 2-brane $\mathscr{O}_C$ wrapping C can be shown to decay as one moves from the large-blow-up limit to the orbifold point in a very similar way to the decay of the 4-branes above [33, 16]. However, the D-brane $\mathscr{O}_C(1)$ can be shown to be stable in the neighbourhood of the orbifold point [34, 16]. Having said that, the rules for orbifolds are not quite the same as for Gepner models. The analysis in the orbifold case is facilitated by the central charges of the B-branes lining up to have the same phase at the orbifold — allowing for quiver methods to be used [34, 16]. This does not happen for Gepner points.

If one can show that all the 4-branes decay on moving out of the geometric phases, then it brings this work into line with the analysis of Bondal and Orlov [17]. They showed how the derived category (i.e., D-branes) could be used to reconstruct an algebraic variety complete with its Zariski topology by using such 4-branes.

On the other hand, if these 4-branes corresponding to nontrivial bundles do tend to remain stable in the non-geometric phases, then we need to motivate why it is the trivial line bundles that are essential in building the Zariski topology.

One might take a completely different approach to determining the target space from the BPS D-brane data. For example, one might say that the target space is given by the moduli space of the 0-brane. Since the 0-brane is stable in the non-geometric phases, one is free to do this. It is not obvious however that this is a perfect definition of spacetime. For example, in [35] it was shown that the metric seen by a 0-brane on an orbifold resolution is not the expected one. I would like to argue that such an approach is essentially different to the algebraically-motivated ideas in this paper. Such a D-brane probe never sees the phase structure anyway since the non-geometric phases get squeezed away as pictured in [28, 35, 36].

If one is wedded to such an idea of a "differential" approach to spacetime geometry, then the ideas in this paper probably hold little appeal. On the other hand, the more "algebraic" approaches to spacetime should always require arguments about stability, such as the ones in this paper, to organize ideas concerning topology.

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